

1 Continuous Time Fourier Transform

Name	Time	Projection	Frequency Equivalent
Even	$x(t) = x(-t)$	$\text{Even}\{x(t)\} = \frac{1}{2}(x(t) + x(-t))$	Even
Odd	$x(t) = -x(-t)$	$\text{Odd}\{x(t)\} = \frac{1}{2}(x(t) - x(-t))$	Odd
Real	$x(t) = x^*(t)$	$\Re\{x(t)\} = \frac{1}{2}(x(t) + x^*(t))$	Hermitian
Imaginary	$x(t) = -x^*(t)$	$j\Im\{x(t)\} = \frac{1}{2}(x(t) - x^*(t))$	Skew Hermitian
Hermitian	$x(t) = x^*(-t) = x^\dagger(t)$	$\text{Herm}\{x(t)\} = \frac{1}{2}(x(t) + x^\dagger(t))$	Real
Skew Hermitian	$x(t) = -x^\dagger(t)$	$\text{SkHm}\{x(t)\} = \frac{1}{2}(x(t) - x^\dagger(t))$	Imaginary

Table 1: Symmetries

Name	Time Domain	Frequency Domain
Synthesis	$x(t) = \int_{-\infty}^{\infty} X(f)e^{2\pi jft}df$	
Analysis	$\int_{-\infty}^{\infty} x(t)e^{-2\pi jft}dt = X(f)$	
Linearity	$\alpha x_1(t) + \beta x_2(t)$	$\alpha X_1(f) + \beta X_2(f)$
DC shift	$x(t) + \beta$	$X(f) + \beta\delta(f)$
Time Shift	$x(t - t_0)$	$X(f)e^{-2\pi jft_0}$
Modulation	$e^{2\pi jft_0}x(t)$	$X(f - f_0)$
Dilation	$x(at)$	$\frac{1}{ a }X(\frac{f}{a})$
Conjugation	$x^*(t)$	$X^\dagger(f)$
Hermitian Conjugation	$x^\dagger(t)$	$X^*(f)$
Time Reversal	$x(-t)$	$X(-f)$
Duality	$X(t)$	$x(-f)$
Differentiation	$\frac{d}{dt}x(t)$	$2\pi jfX(f)$
Frequency Differentiation	$tx(t)$	$\frac{-1}{2\pi j}\frac{d}{df}X(f)$
Convolution	$(x * y)(t) = \int_{-\infty}^{\infty} x(\tau)y(t - \tau)d\tau$	$X(f)Y(f)$
Multiplication	$x(t)y(t)$	$(X * Y)(f)$
Correlation	$\rho_{x,y}(\tau) = \int_{-\infty}^{\infty} x(t)y^*(t - \tau)dt = (x * y^\dagger)(\tau)$	$X(f)Y^*(f)$
DC value	$\int_{-\infty}^{\infty} x(t)dt = X(0)$	
Energy	$E\{x\} = \int_{-\infty}^{\infty} x(t) ^2dt = \int_{-\infty}^{\infty} X(f) ^2df = \rho_{x,x}(0)$	
Parseval's Formula	$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df = \rho_{x,y}(0)$	

Table 2: Continuous Time Fourier Transform Properties

Name	Time	Frequency
impulse	$\delta(t)$	1
constant	1	$\delta(f)$
box, rectangle	$\Pi(t)$	$\text{sinc}(f)$
<i>sinus cardinalis</i>	$\text{sinc}(t)$	$\Pi(f)$
Gaussian	$e^{-\pi t^2}$	$e^{-\pi f^2}$
causal exponential	$e^{-t}u(t)$	$\frac{1}{1 + 2\pi jf}$
signum	$\text{sgn}(t)$	$\frac{1}{\pi jf}$
unit step	$u(t)$	$\frac{1}{2\pi jf} + \frac{1}{2}\delta(f)$

Table 3: Fourier Transform Pairs